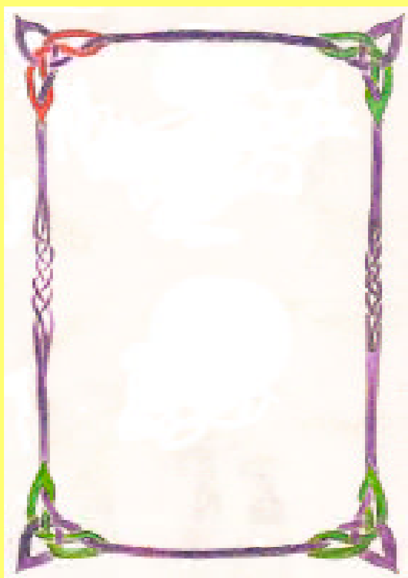
Geometry Lessons in the Waldorf School

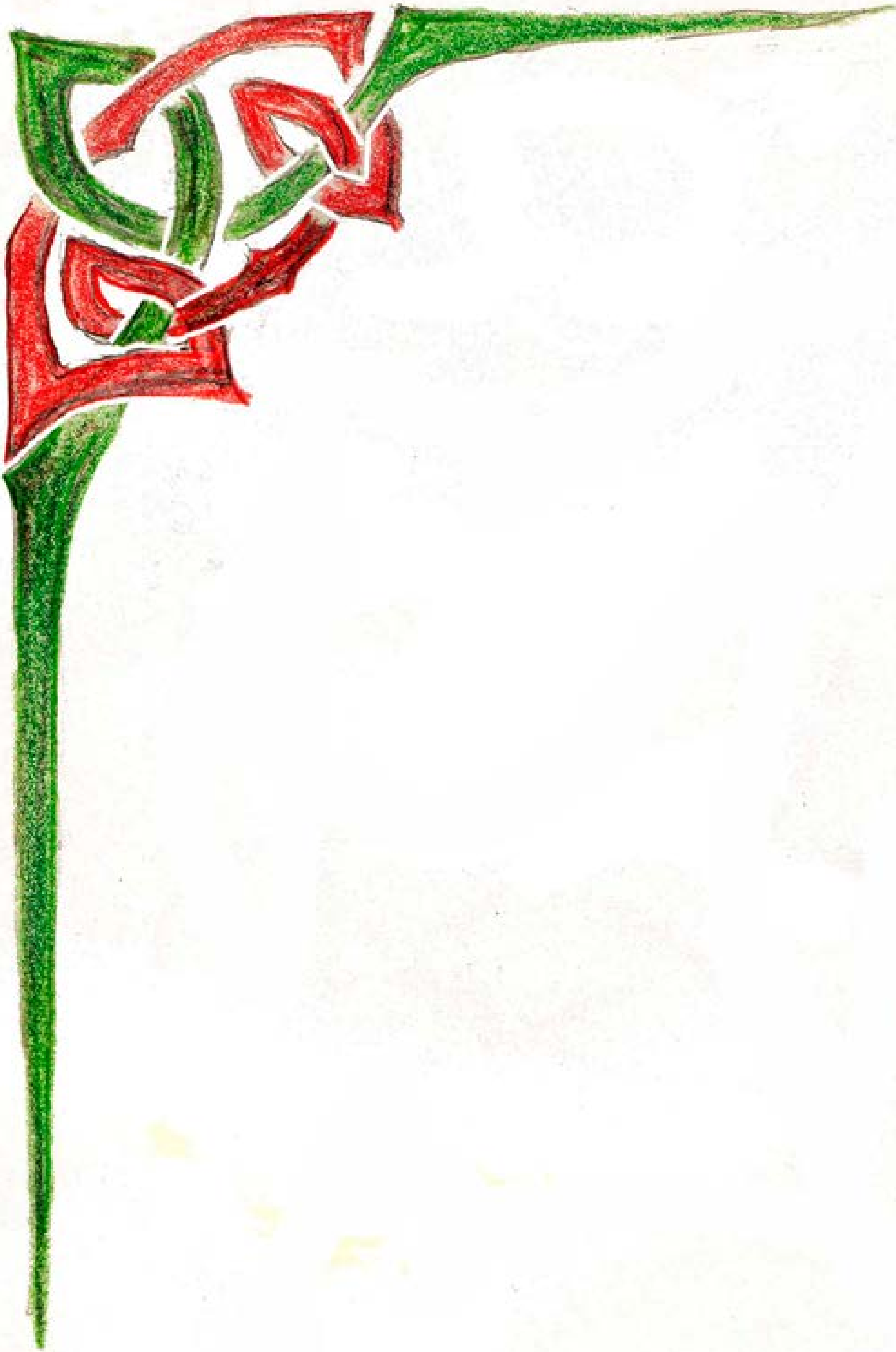
Volume 2: Freehand Form Drawing and Basic Geometric Construction in 4th and 5th Grades

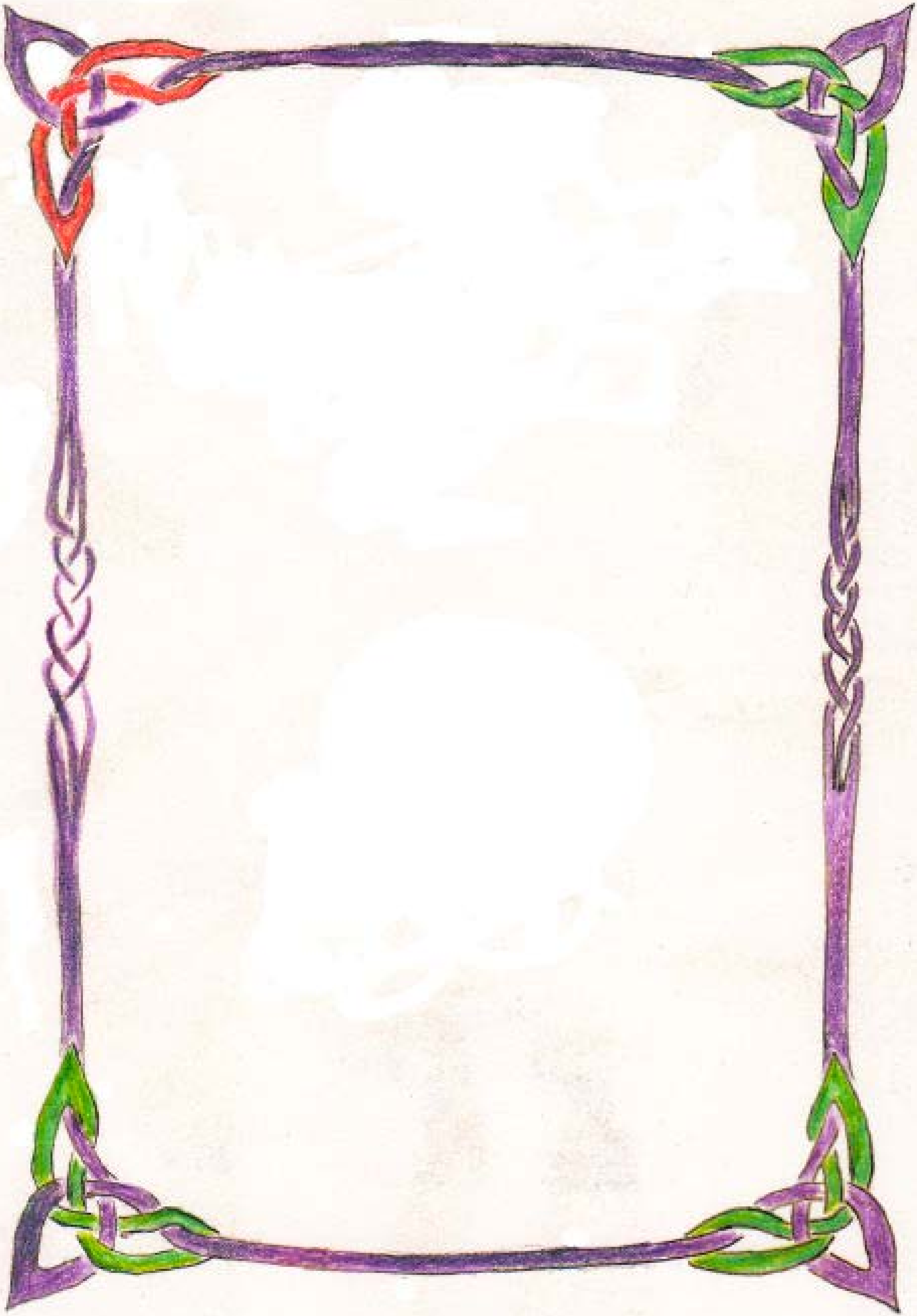
Book Design Forms



by

Ernst Schuberth





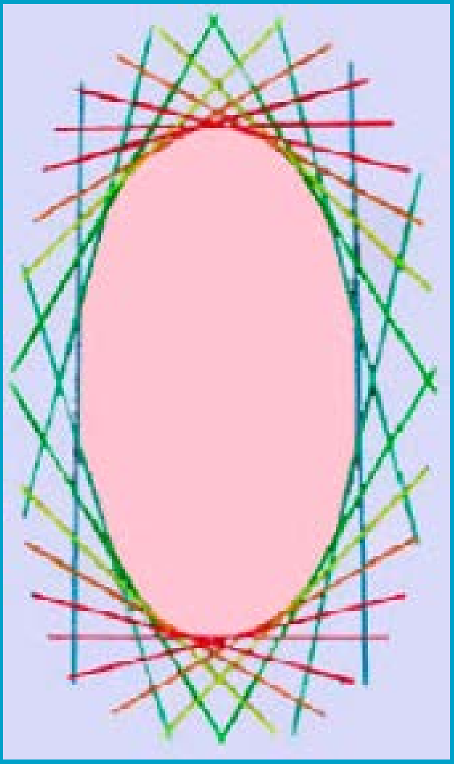
Geometry Lessons in the Waldorf School

Volume 2: Freehand Form Drawing and Basic Geometric Construction in 4th and 5th Grades

Grade 4

by

Ernst Schuberth



Companion CD Rom to:
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Author: Ernst Schuberth
Editor: David Mitchell
Proofreader: Ann Erwin

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1. Geometry in grade 4
2. Geometry in grade 5
3. Two examples of student main lesson books
4. All drawings as jpg and tif files for you to enlarge and study
5. Some knotted forms for the fourth grade

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I look forward to getting a responses from you. Did you develop new beautiful exercises? Did your students work out drawings very carefully? Could any part of the book be more understandable? Did it work in your classroom? Please help me and all users to improve our geometry teaching. My email address is:

Ernst.Schuberth@t-online-de

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— Ernst Schuberth
Mannheim, 2/12/2004

Introduction

There are two main themes to bring into the classroom when we begin geometry in grades 4 and 5: the importance of movement and the idea of space and counterspace. The priority of movement comes from the fact that any form is always created by movement. We cannot create or recognize a form without an inner or physical movement (of our eyes, hands, or other parts of our body). We can also develop an inner flexibility by transforming one form into an other. Many forms are related to each other by a transformation or metamorphosis. That is why we introduce the triangle and other forms out of a circle.

The idea of space and counterspace was first described by Rudolf Steiner; George Adams and Louis Locher-Ernst then worked out his ideas in terms of projective geometry. Although projective geometry is a high school subject, the concept behind projective thinking can be brought much earlier. The physical forces work mainly in well-known space; life forces have a relation to the idea of counterspace. Space as we usually think of it in physics has to do with central forces, while counterspace is related to spherical forces which work from outside to inside.

Geometry in Grade 4

Colored Drawings, Supplementary Exercises, and Additional Notes

by

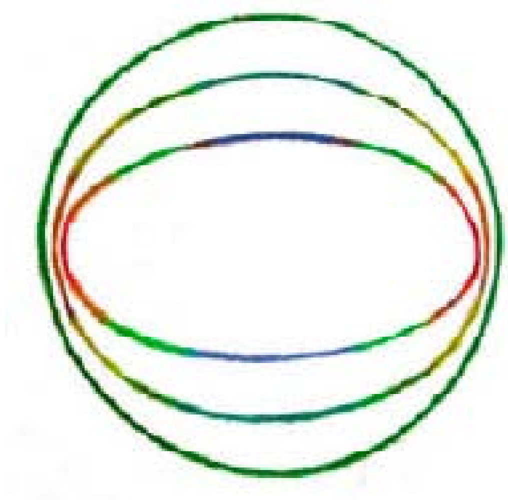
Ernst Schuberth

Exercise 1: From the circle to the ellipse

Movement is the origin of geometry. Whenever we perceive a form we move at least our eyes. Years ago I taught a blind student. To understand any geometry she needed help from a classmate who drew the form into her hand. With her intention she followed inwardly the movement, creating an inner movement within herself. This is very important. It is not enough to move the children. They must internalize this movement by creating an inner picture.

Please read the story about John in my book. It illustrates some important details. Picture the whole process on the blackboard and draw on the blackboard how he moved. Then address the feelings by using colors. I did it in the following way: We started with a circle, a calm and peaceful movement. Then, walking the circle we changed the form slowly into an ellipse. On both ends we needed more activity because of the stronger curvature. This is expressed by the colors: More rotation at the ends, more calm walking in between.

Drawing 1: Calm movement on a circle line

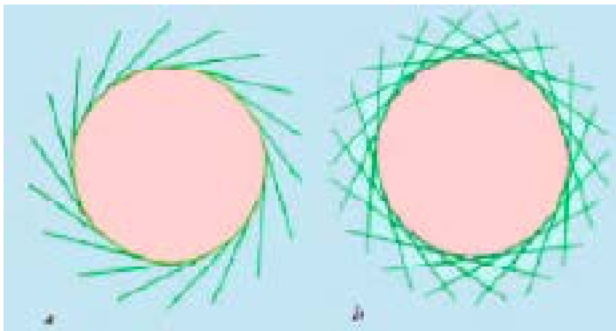


Drawing 2: From the circle to the ellipse

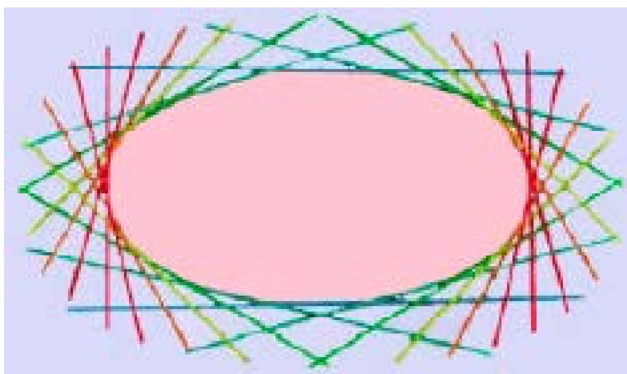
Exercise 2: From the circle to the ellipse

We continued to work on Exercise 1. In order to bring more consciousness into the two types of movement we showed the direction at each point with a stick. The stick is the tangent although we did not define this word.

The following drawings show how I brought in the idea of a line-wise curve. Usually we always underline the aspect of a form as a set of points. Again the color describes the activity of a stronger rotation at the ends.



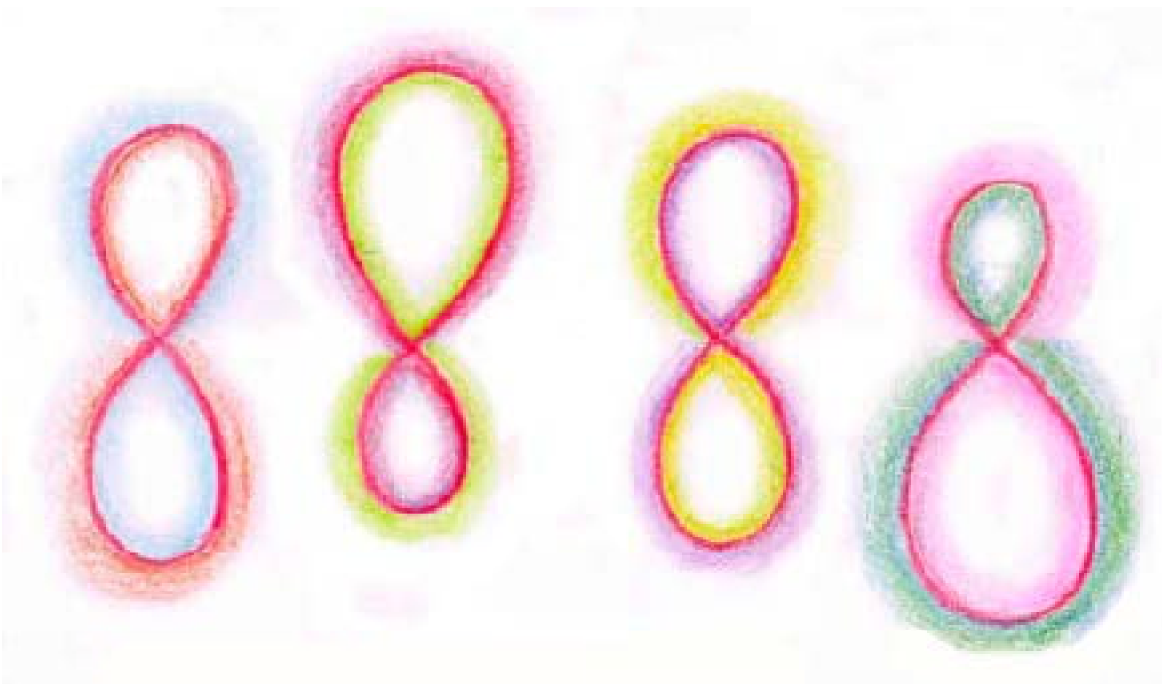
Drawing 3: The movement on a circle and its directions



Drawing 4: More change of the directions at the ends of an ellipse (red)

Exercise 3: The breathing rhythm of day and night during the year

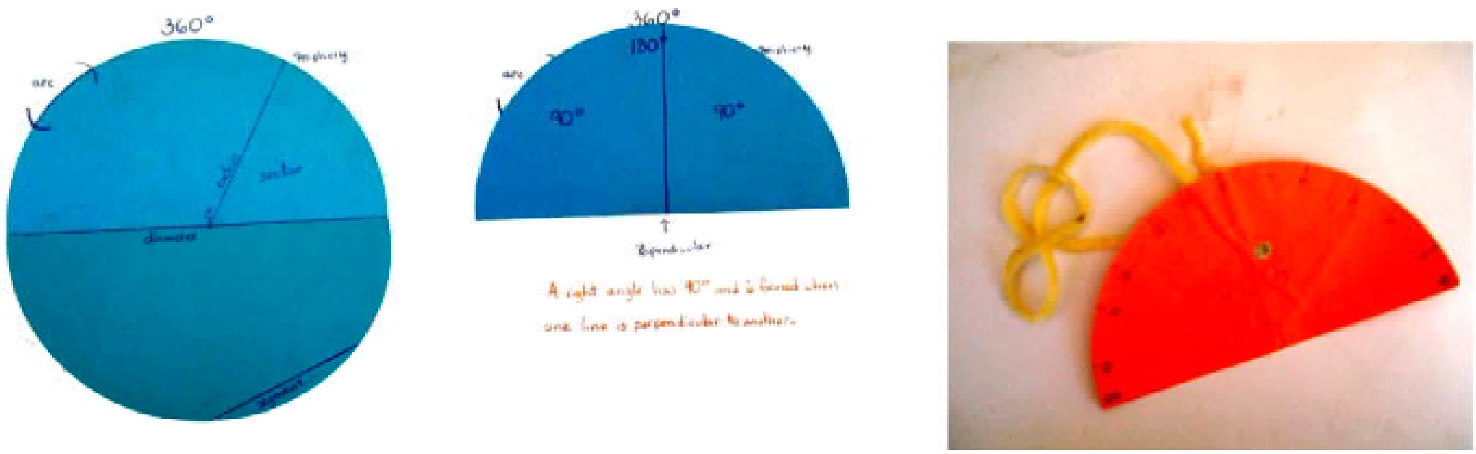
While introducing the measurement of angles, take the opportunity to give a lively description of the cosmic rhythms and our use of angles. The four seasons have differing lengths of day and night. As described in the book, in former times the time from sunrise until sunset was divides into twelve hours. The same happened with the night. So the “hours” had different lengths depending on the season. The lemniscate below pictures this change during the year. The upper part characterizes the day, the lower part the night.



Drawing 5: The breathing rhythm of day and night during the year

Exercise 4: Construct your own protractor

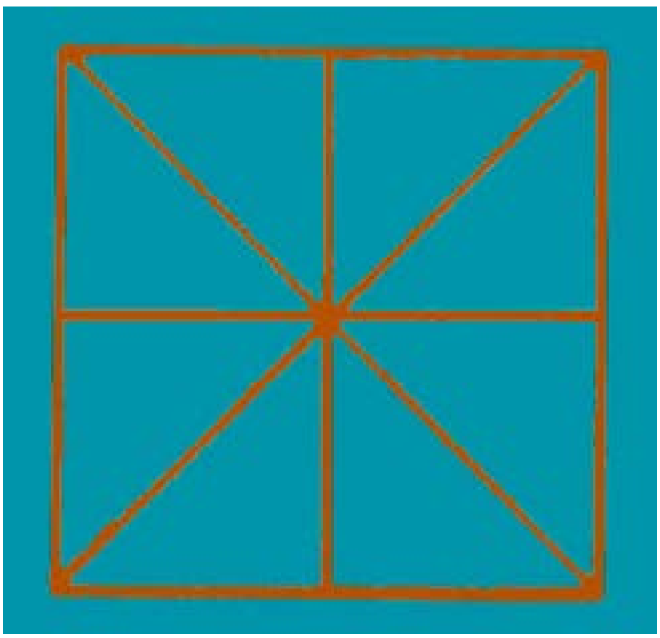
Having learned that the full circle – taken from the zodiac – is divided into 360°, the students can construct their own protractors. Start with cutting out a circle. Fold it and get the first subdivision. Bring it into relation with fractions (introduced before this block). The photos below show examples from a student's work.



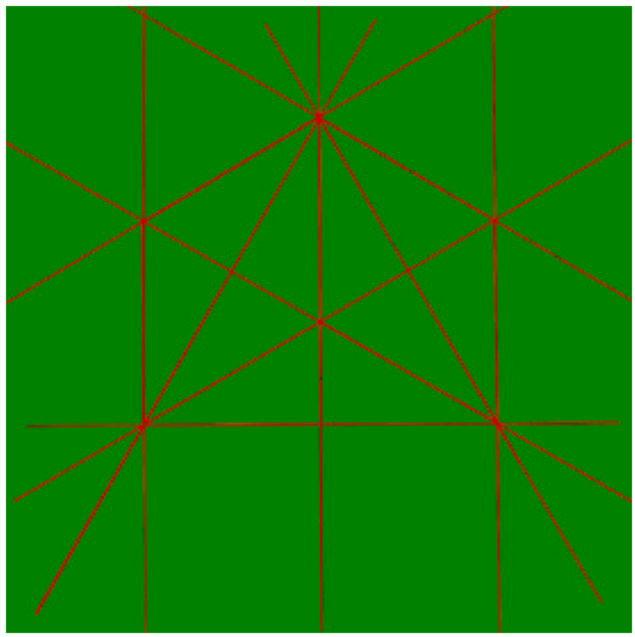
Drawing 6: Student's first protractor

Exercise 5: Find angles

Find angles in the following drawings. Estimate and measure. (On the left side you will find 45° and all multiples of it; on the right side 30° and all multiples.)

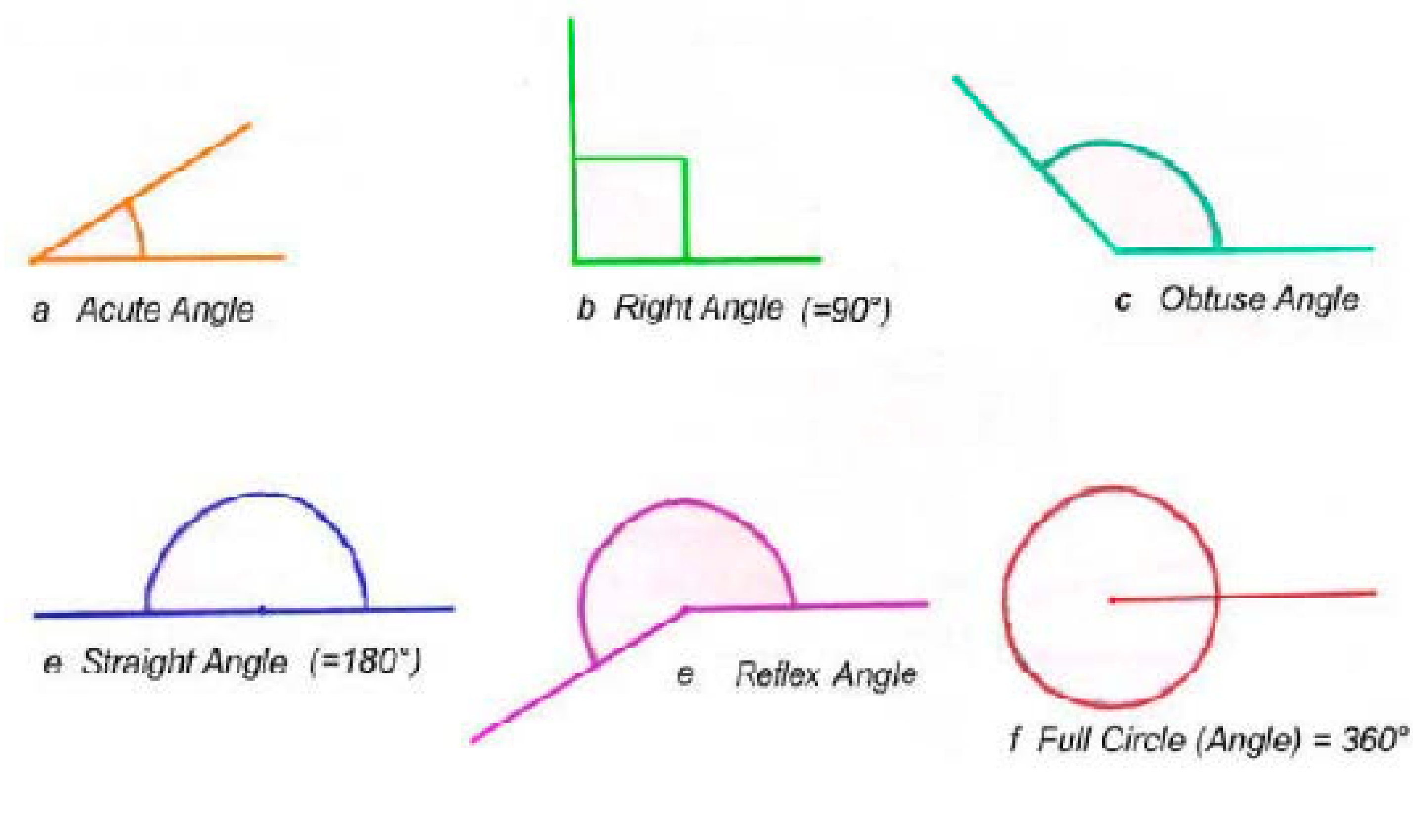


Drawing 7: Find angles 1



Drawing 8: Find angles 2

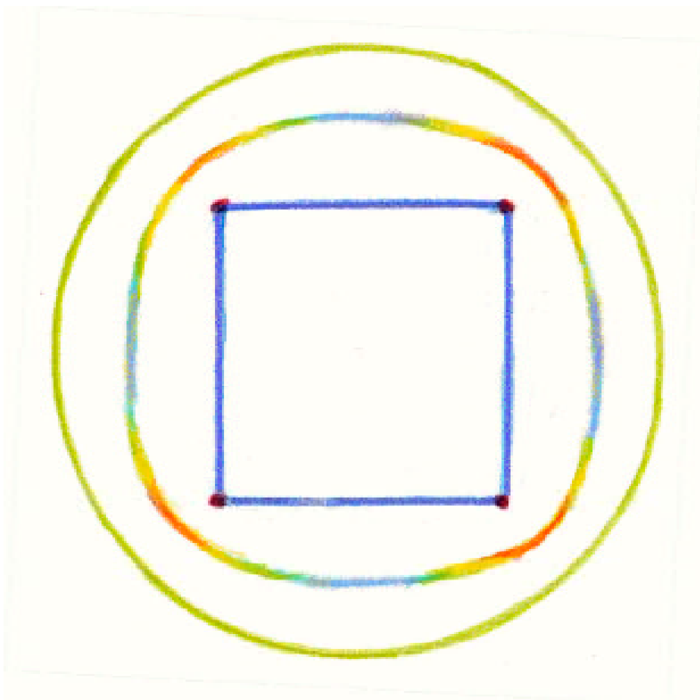
Show angles of the different types with your arms. Find them in the following drawings.



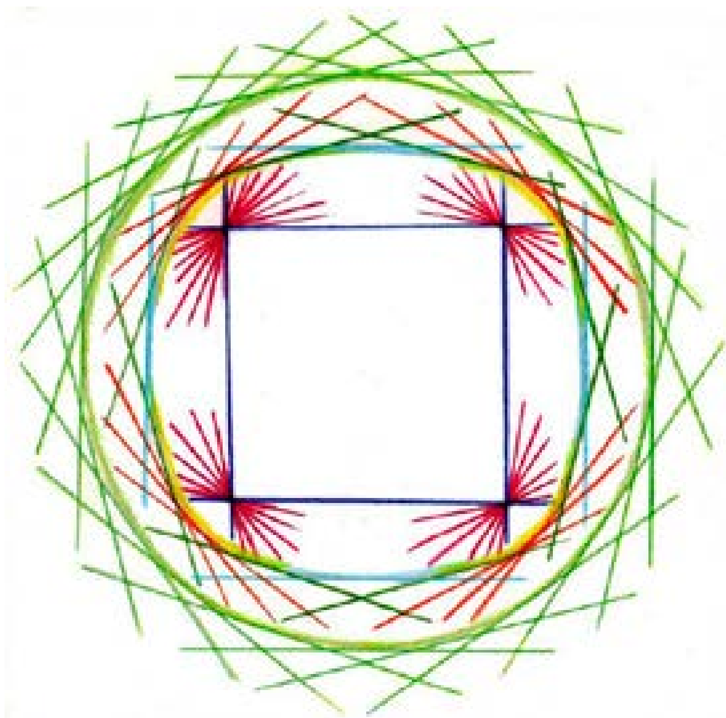
Drawing 9: Different angles

Exercise 7: The triangle as a metamorphosis of the circle

Start with a walking exercise. Walk on the circle and transform it slowly into a triangle. Draw three situations on the blackboard (Drawing 10).Add the tangents to make the different directions visible (Drawing 11). Discuss how more and more the rotations concentrate at three points. In the triangle walking and rotation fall away. It is like a death process.



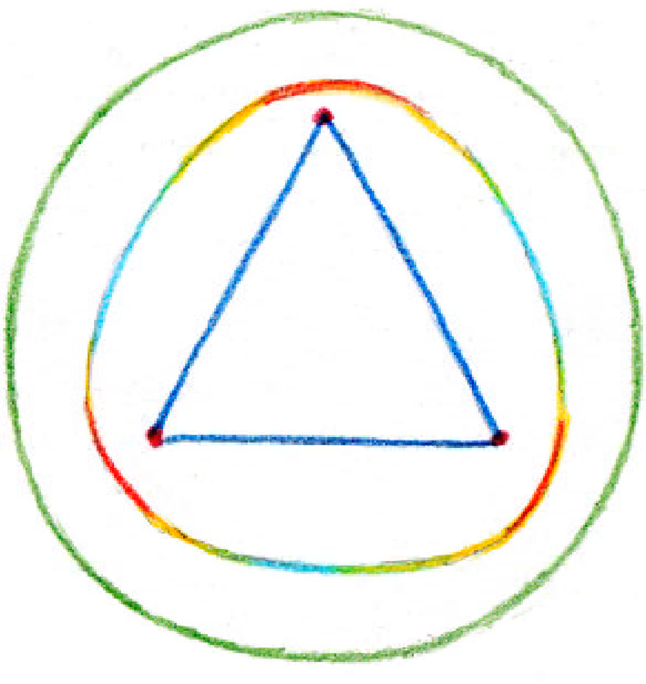
Drawing 12: From the circle to the square 1



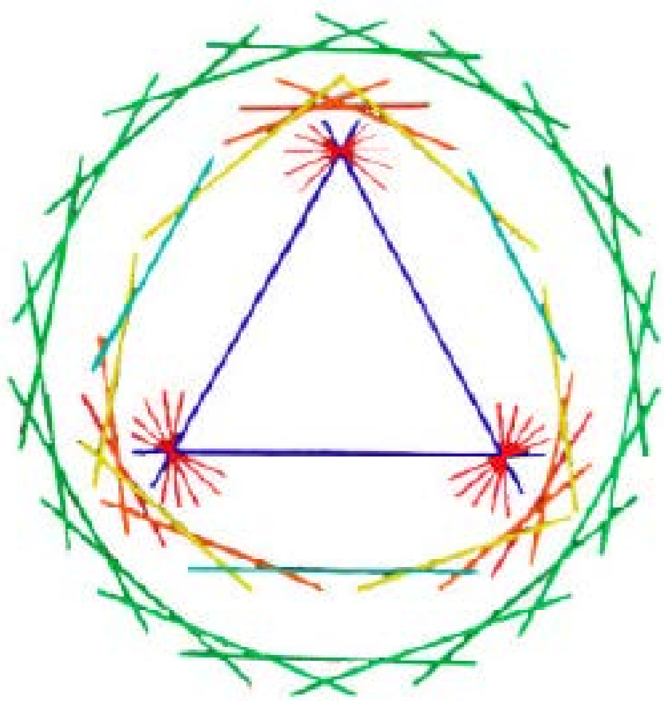
Drawing 13: From the circle to the square 2

Exercise 8: The metamorphosis of a circle into a square

Start with the circle as we did with the triangle. Walk on the circle line and transform it slowly into a square. Stop at the corners of the square. Rotate only. Let the students observe and describe the change. A rhythm develops between walking (translation) and rotation. Draw three stages on the blackboard. Then add the tangents in order to characterize the changing directions. Again they should feel that during this metamorphosis, the two kinds of movement fall more and more away.



Drawing 10: From the circle to the triangle 1

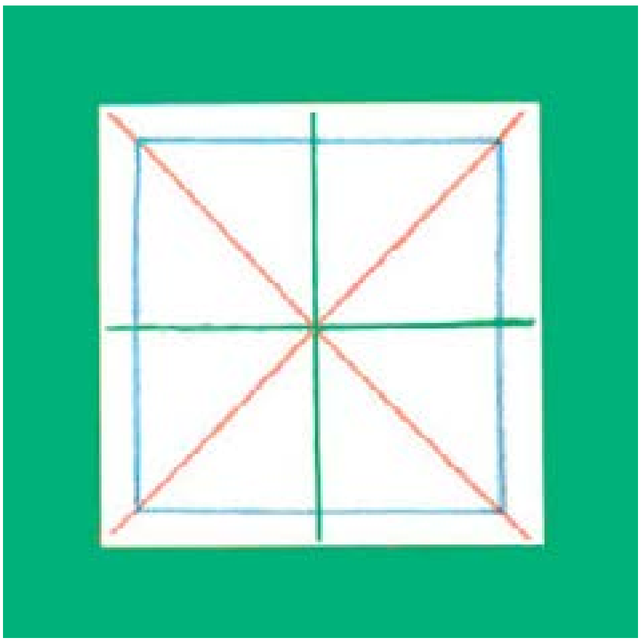


Drawing 11: From the circle to the triangle 2

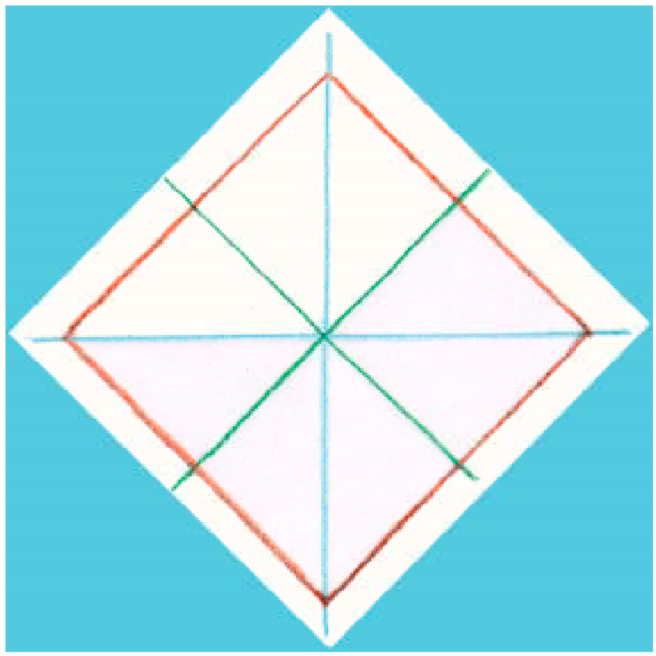
Exercise 9: Squares

Start with a square. It is the most symmetrical quadrilateral. All other quadrilaterals are derived from it by the loss of symmetry. We should always start with an archetypal form, not with the most general concept.

For the students of this age the two positions shown below are essentially different. The left one seems to be heavier, the right one seems to soar. Remember: The teacher has to develop the feelings, not an abstract understanding. Ask the students how they experience the two squares. Then discuss the axes of symmetry. You may cut them out and fold them.



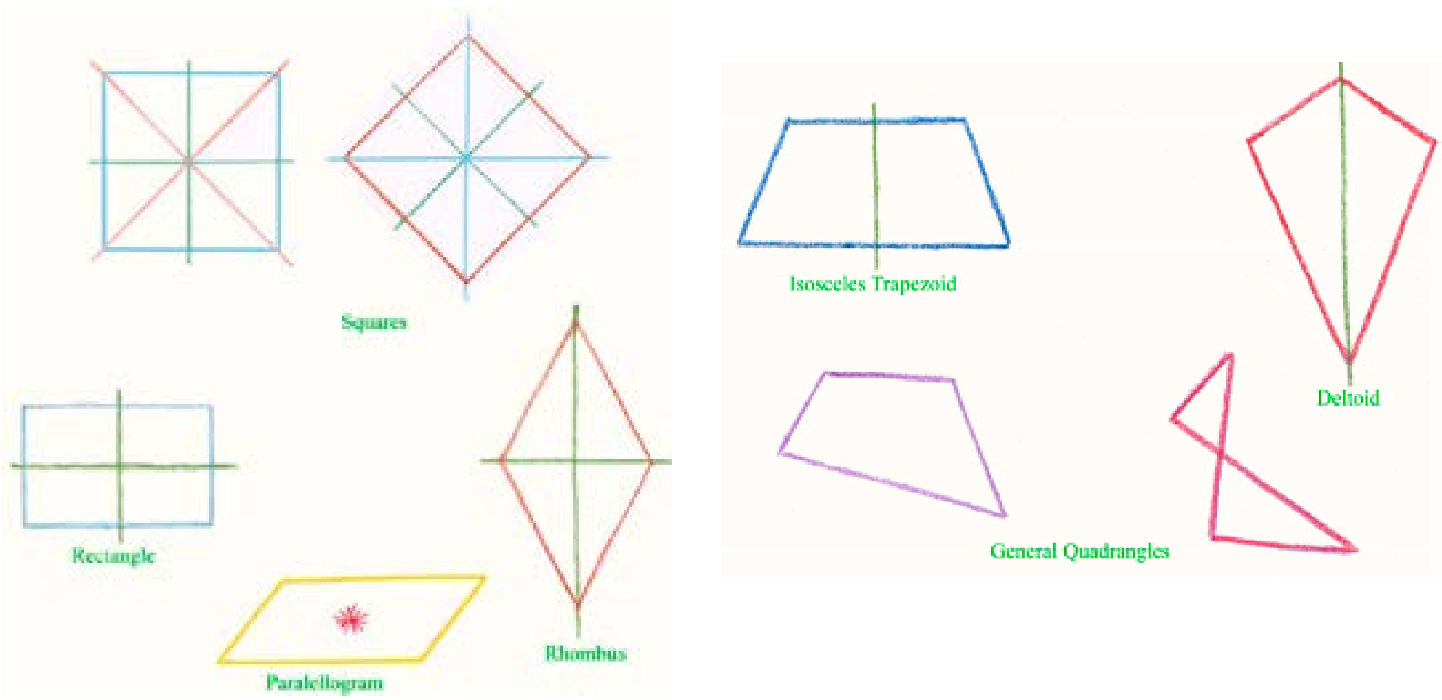
Drawing 14: A square and its symmetry 1



Drawing 15: A square and its symmetry 2

Exercise 10: Quadrilaterals

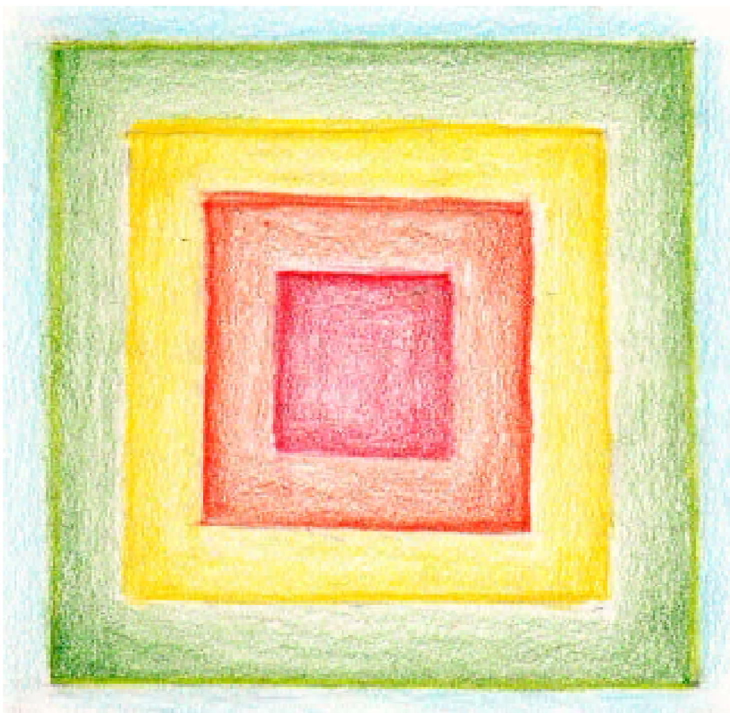
Give a vivid description of all the different quadrilaterals. Let the students feel how, step by step, the quadrilaterals lose symmetry. In my book is a story for the whole family. Describe their relationship rather than give separate definitions of the single form.



Drawing 16: The House of Quadrilaterals

Exercise 11: Growing square

You will find in the book a lot of freehand exercises. As examples of how they can be done by the student see the example below.



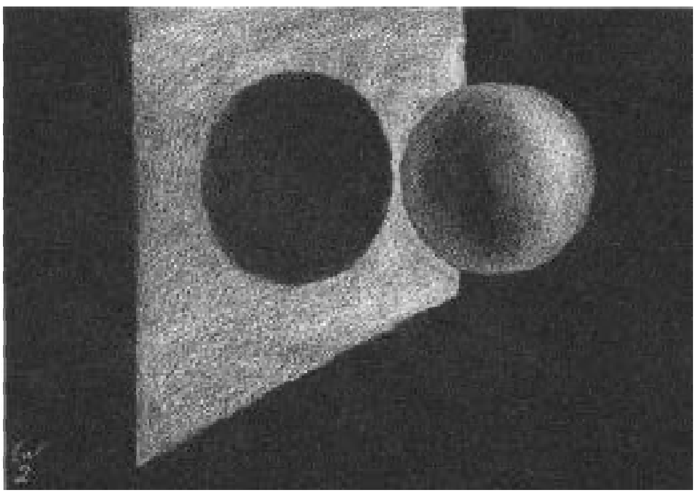
Drawing 17: Growing square

Exercise 12: Observation and discussion of a shade

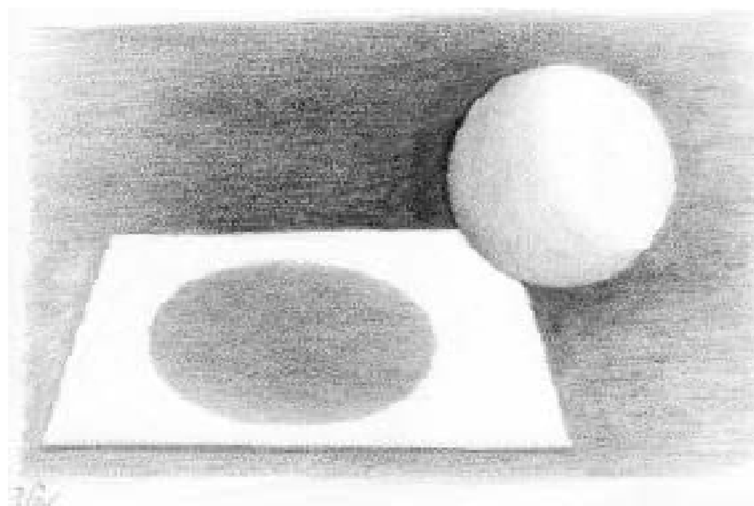
It is an stimulating exercise for students to understand the coming into being of the different forms of a shade. How can a sphere, which looks round like a circle from all directions, create an elliptical shadow? Use a sphere, the sun, and a sheet of white cardboard.

Here we have a wonderful opportunity to develop a spatial imagination. Imagine, we can not see whether a space is full of light or dark. Only the surface of matter tells us whether there is light or not. Around a source of light we have the potential to make matter visible. If there is a not translucent body, behind the body we have a space of darkness surrounded by a space of light. If the light comes from the sun this space is more or less a cylinder. Penetrate this cylinder with a white sheet of paper in different positions and you will see different shadows.

My colleague Elisabeth Wannert in Switzerland has given me two drawings which are much better than my drawings in the book. They also appear in my book on sixth grade geometry avasilable from AWSNA Publications.



Drawing 18: Shade of a sphere 1



Drawing 19: Shade of a sphere 2

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Grade 5

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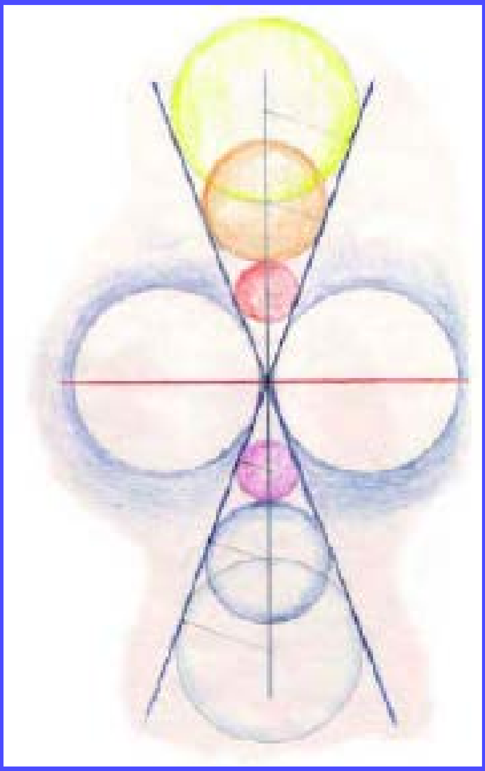

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Geometry in Grade 5

Colored Drawings, Supplementary Exercises, and Additional Notes

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The Fifth Grade

Overview

In the introduction I underlined the importance of movement in geometry and the idea of space and counterspace. When we come back to geometric drawing in the fifth grade we can work with both ideas by introducing the circle again out of movement but now from another perspective. We will describe the circle as a meeting of two currents—something like a surf line. We start with two movements: one that widens a form and another that envelops it. This leads to the polar aspects of the circle: as a kernal or as a hull. We can then describe the different relationships of points and lines with the circle. The next step leads to the symmetry of one and two circles and some additional special cases. We can introduce freehand exercises to finish this lesson.

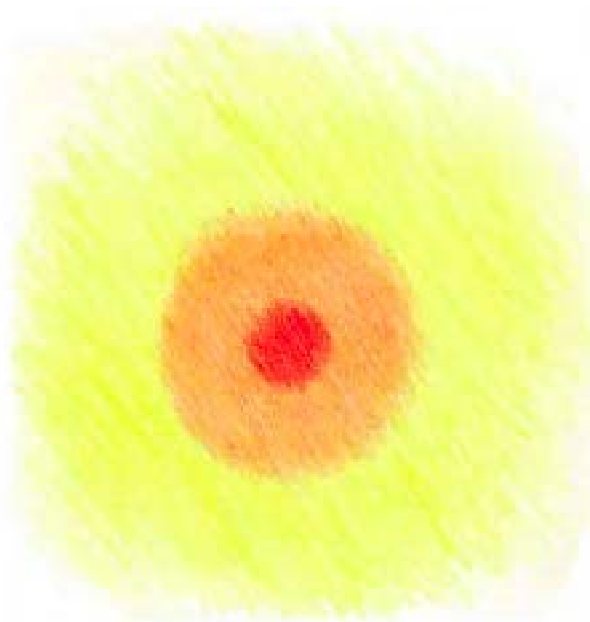
The second part brings the introduction of the ruler and compass. Toward the end of fifth grade the students are ready to handle these instruments. My recommendation is to take at least two weeks, not only to practice exercises but also to discuss fundamental constructions such as the perpendicular midline, the construction of the middle of a segment, and others. Whenever a colleague has followed these suggestions, he has received enthusiastic feedback from his students. Let me know your experiences. If you are not able to deal with all the fundamental constructions this not really a problem. Most important is to deal more consciously with the geometric forms. After freehand drawings which can be most challenging (for example, a circle), it seems suddenly very easy to construct them with tools. And indeed we train less of the children's constitution with geometric tools than we do with freehand drawings. But the step has to be taken, and we will come back to freehand drawing again and again (although I do not describe this in my book).

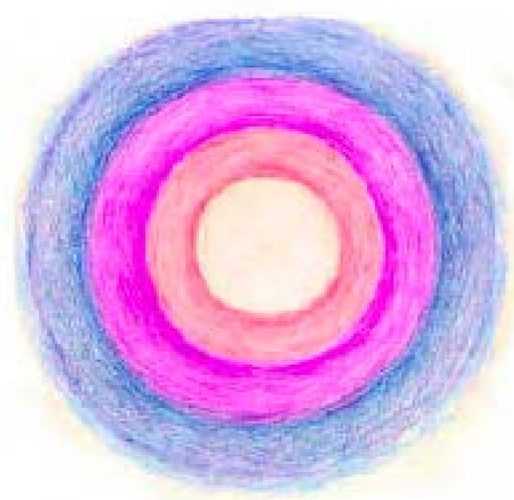
At the end of this booklet you will find a first brief look at the Pythagorean Theorem. Rudolf Steiner recommended to teachers that they start with the isosceles right-angled triangle.

Exercise 1: Circles growing from inside and outside

Start with yellow and let it widen from inside to out. Then take orange and strengthen the center. Let it widen too but not as much as the yellow. Take red and do the same. Find a good balance between the quantities of the three colors.

Start parallel with a light blue from outside and let it grow toward the center. In both cases the form should always be a circle. In the first case we have a growing out of the center (space), in the second case a growing from infinity toward the center (counterspace). The teacher'scareful description of the whole process is very important. The children will be able to understand that what one feels is important.

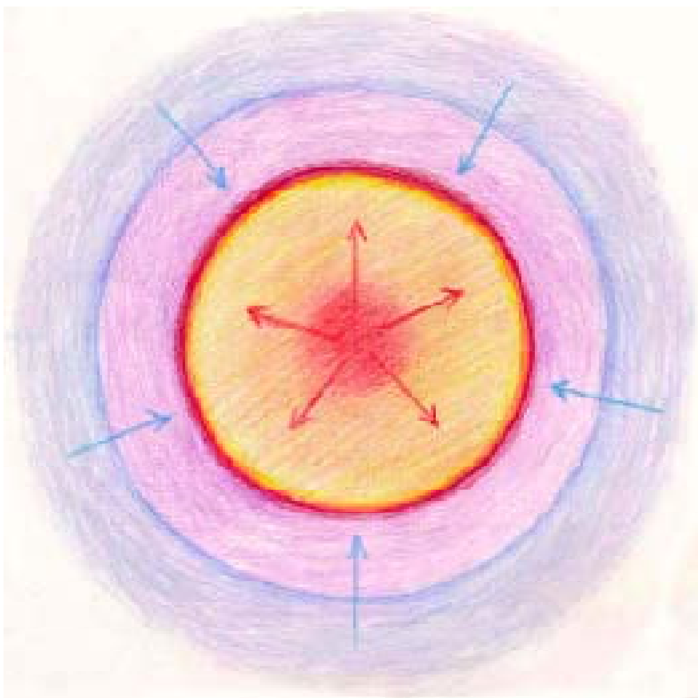




Drawing 1: Growing circles from inside and outside

Exercise 2

Now give a very dramatic description of a surf line where two currents meet, one from the beach and the other from the vastness of the the ocean. Here we have the currents from inside (red and yellow) and from the outside (blue and violet). Where they meet they create a form, depending on the strength of the two forces. If you have little time, create the circle as the direct encounter of the two currents. When you feel that both you and your class have developed a strong feeling for this drama, go deeper into it. (See following exercises.)



Drawing 2: The circle as surf line

Exercise 3

Here the inner and the outer forces fight against each other in different ways. Start with a circle and give a pictorial and dramatic description of the fight between the inner and the outer forces. Let the children find what the result will be if the inner forces are stronger (left side) or the outer forces are stronger (right side). Discuss this polarity.

Drawing 3: Iinteraction between inside and outside

Exercise 4

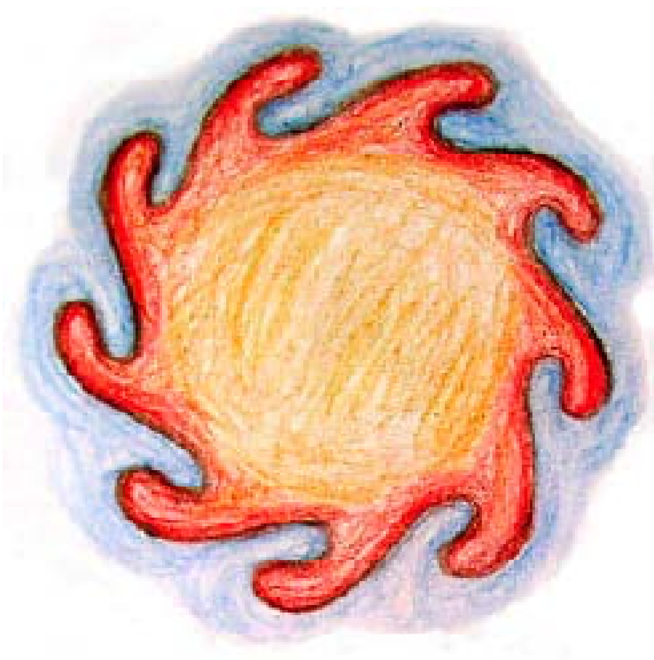
This exercise helps develop an artistic feeling for processes of form. When the inner and the outer meet as described in exercises 2 and 3, one of them can be stronger than the other.

Drawing 4: Relations between inside and outside

Interesting point to consider: The concave form with the weaker inner force looks more aggressive than the convex one. Aggressiveness is often an expression of inner weakness.

Exercise 5

The interaction between the inner and the outer can also be rhythmical: the outer comes in and the inner streams out. The figure seems to rotate. Be aware of the relation between the inner and the outer. In the left drawing the inner is not very penetrated by the outer. It is not much moved by the influence from outside. The right drawing shows a more lively relation. Let the children find their harmony and then compare different results.

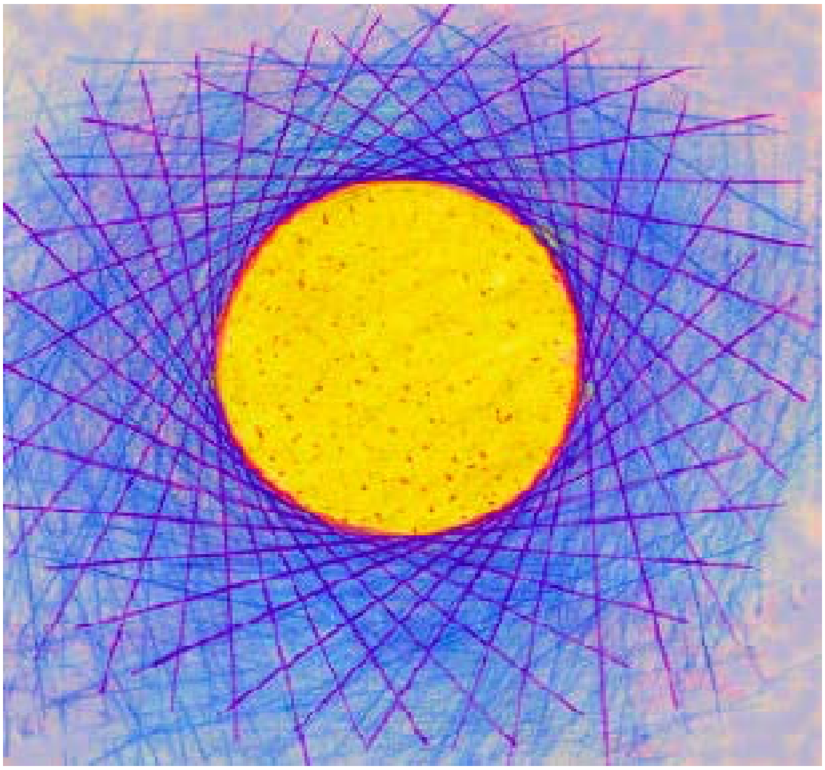


Drawing 5: Rhythmical interaction 1

Drawing 6: Rhythmical interaction 2

Exercise 6

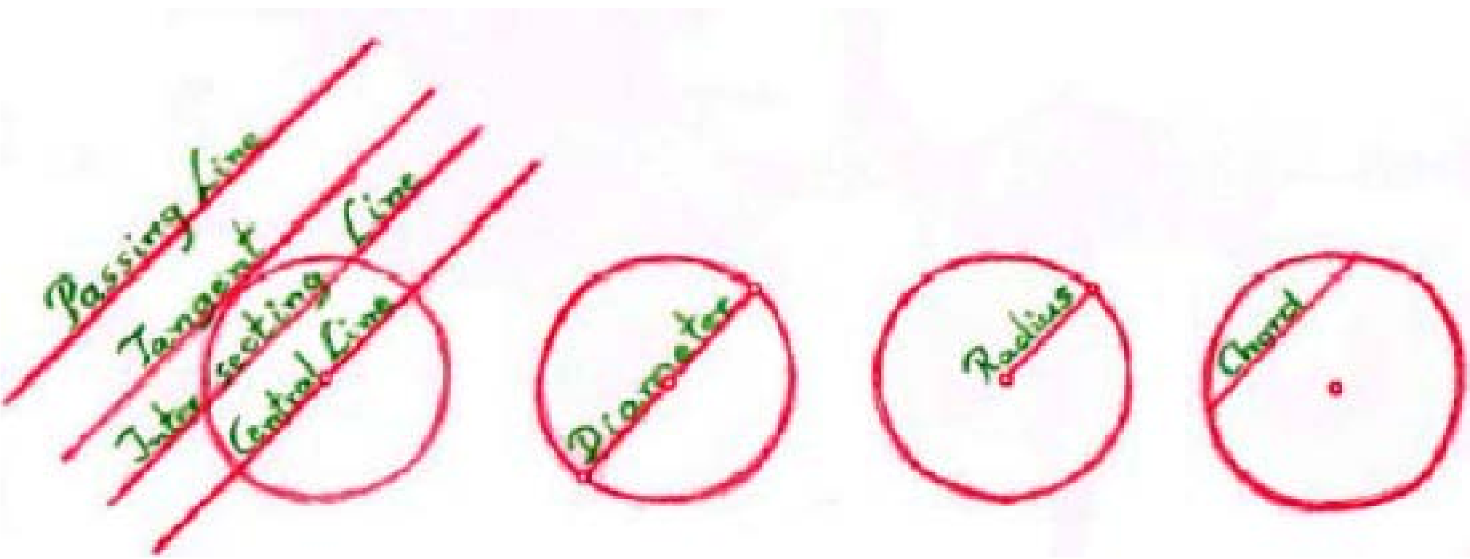
When we try to shape the polarity of inner and outer in terms of geometry we can use lines and points: the outer can be shaped by lines – forming the hull of the circle – and the inner by points – forming the kernel of the circle. These are polar pictures. The drawing below shows them both. But remember that there are also points outside the circle and lines passing through the circle. We will now describe the different types of lines and points in relation to the circle. (See next pages.)



Drawing 7: Hull and kernal of a circle

Exercise 7: Circle and lines

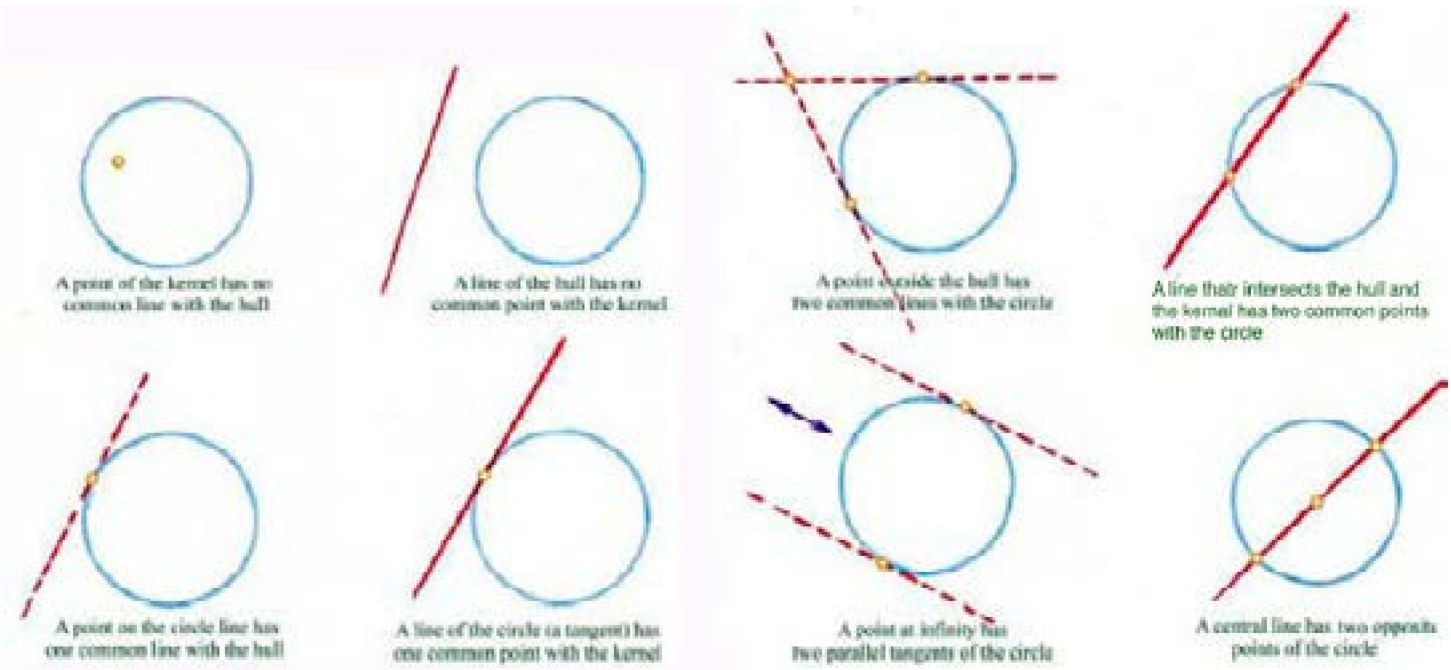
Lines have different relationships with a circle depending on how many points they have in common with the circle's arc: none, or one, or two. Discuss the different names and ask for better ones. Derived from the intersecting line we learn as special cases the central line and special segments: the diameter, the radius, and the chord.



Drawing 8: Circle and lines

Exercise 8: Circle, line, and point

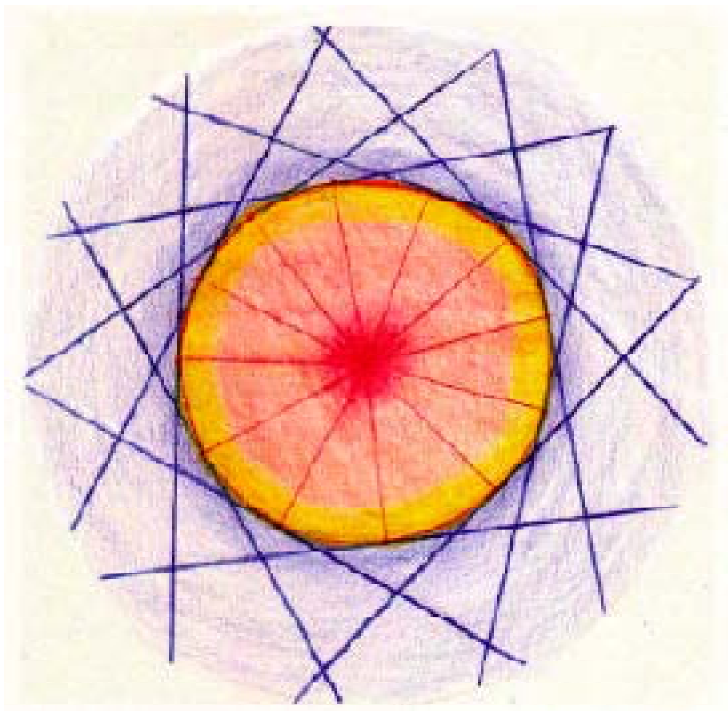
In this drawing you see again a circle with points and lines. In the book you saw different lines in relation to the circle depending on the number of common points with the circle. In a polar way you can distinguish the relation of a point to the circle depending on the number of common lines with the circle. They may be none, or one, or two (left side). Ask the children: “What happens if the point is very far away, if it is at infinity?” Some of them will be able to answer this question, although they will get an introduction to projective geometry later in the high school. The two tangents through a point at infinity are parallel. The line at infinity – as mathematicians call it – is the polar counterpart of the center in a circle.



Drawing 9: Circle, point, and line

Exercise 9: Points and lines of a circle

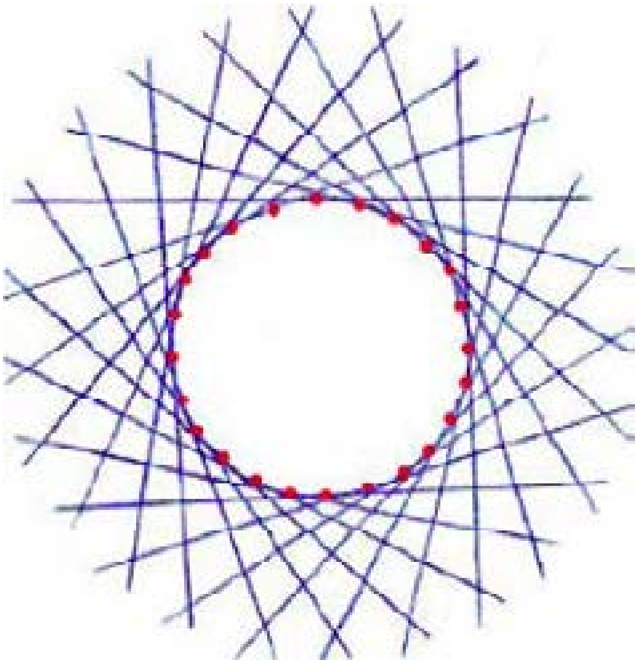
This drawing shows again the polar aspects of a circle. We can look at it in a point-wise or line-wise way. The points on the circle line have different places. The lines of the circle which we call tangents have different directions. When a point walks on the circle it moves always in the direction of the assigned tangent. When a tangent moves around the circle it rotates around the assigned point. Everywhere a line and a point belong together. Measured from the center of the circle all points are equidistant. We call this segment a radius of the circle. Tangent and assigned radius always form a right angle.



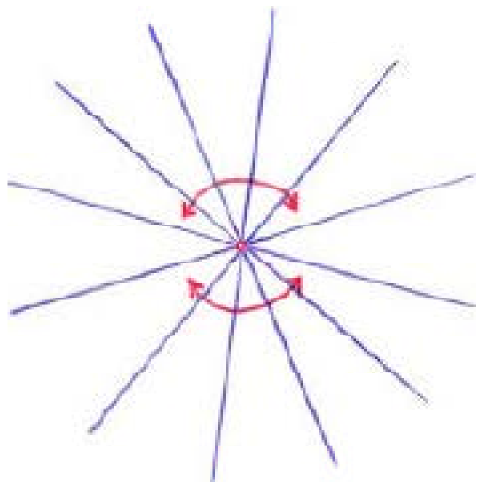
Drawing 10: Lines and points of a circle

Exercise 10: Transformation of a circle into a point and a line

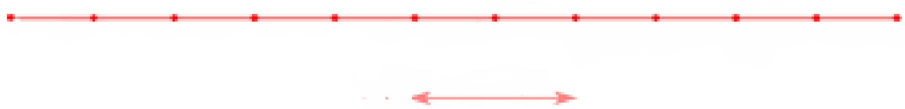
A circle with its points and lines can degenerate in two ways. It can shrink until it is only a point with an infinite number of lines. We call this a pencil of lines. A line rotates in a pencil. When, through polar growing, the circle becomes larger and larger until it is a straight line, this line has an infinite number of points. We call this a row, or a range, or a line of points. Choose one expression. A point translates on a line of points. Rotation and translation are polar movements.



Drawing 11: A circle with its points and lines



Drawing 12: A pencil of lines

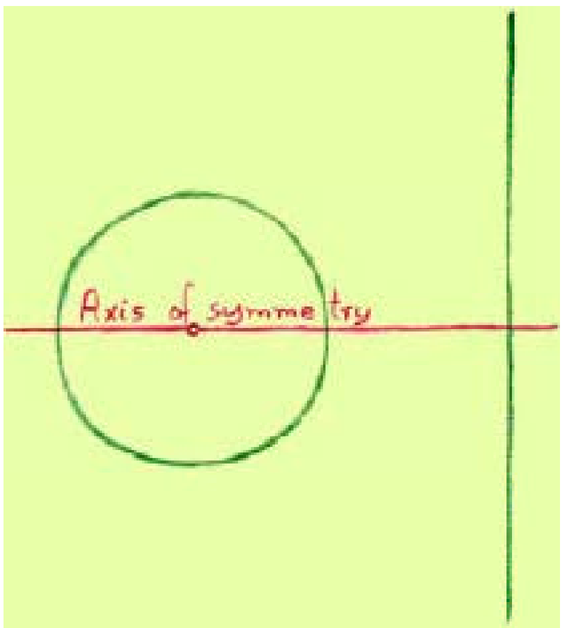


Drawing 13: A line of points

Exercise 11: Symmetry of one or two circles

In the book you can find a section about symmetry in connection with circles. These considerations are used later for constructions and proofs. Do not spend much time with them; make freehand drawings, although I used ruler and compass for them. For the children it will be helpful to have different circles cut out of cardboard. Use big ones for your teaching. Start with the cardboard circle. Ask for the axes or lines of symmetry. Make a freehand drawing on the blackboard and draw what the students found.

Transforming a circle again and again into a point or a line is stimulating. Drawings 59 and 60 show examples for such a transformation. What happens with the symmetry in these extreme situations (or borderline) cases? The drawing below shows a one-colored example of the symmetry of a line and a circle.



Drawing 14: The symmetry of a circle and a line

Exercise 12: Growing circles

Drawing 59 can be called a growing seed. It is a transformation of a point into a line. Start with a point on a horizontal line. Let it grow until it becomes a line. Give an enthusiastic description of this infinite growing. No paper is large enough to show all circles. Let the children find a good proportion for each step. Drawing 60 is still more complicated. The students should find the inner proportion by their feeling. In the high school they will learn more consciously about the Circles of Apollonius.

Drawing 15: Growing circles

Exercise 13: Logarithmic spirals

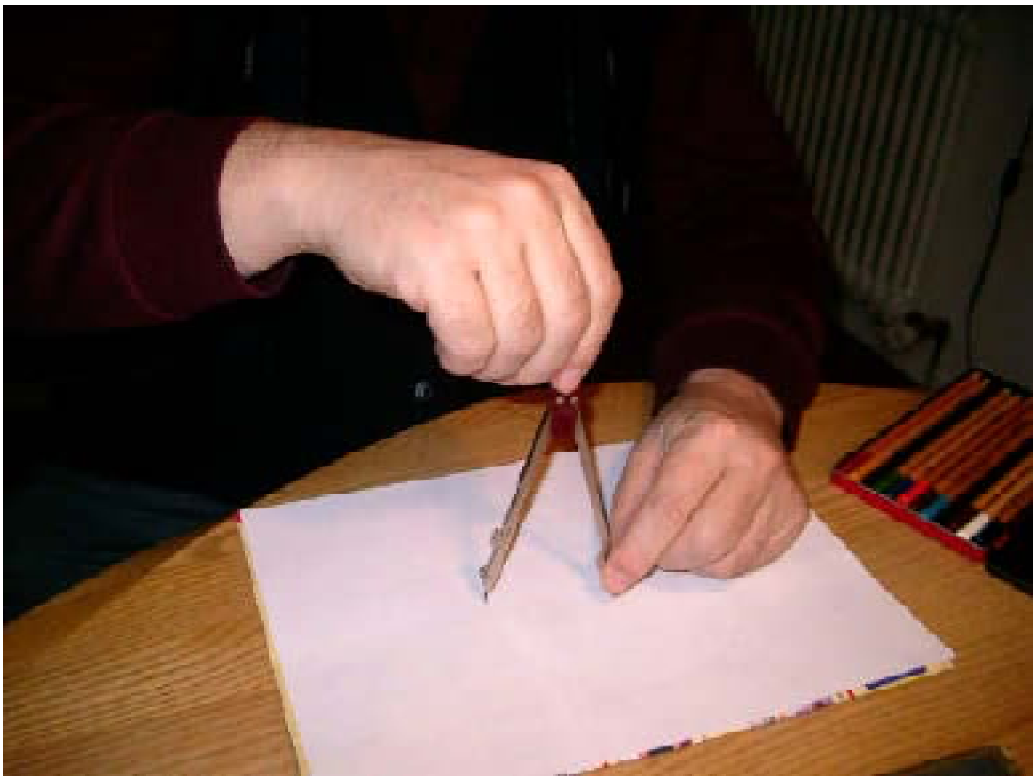
All exercises in this chapter should be done with colors. Drawing 65 was already suggested for the fourth grade (Fig. 31). If the students are capable, show them how a similar drawing can be done with a pentagon and a hexagon. Start with a lead pencil and draw the pentagon or the hexagon. Find the center. Draw some auxiliary lines from the center to the points and from the center to the middle of the sides. The students will get an inner feeling for the logarithms which are hidden in the spirals.

Drawing 16: Pentagon spiral

Drawing 17: Hexagon spiral

Remark

As I described in the book ruler and compass should be introduced carefully. Show the students how straight the edge of the ruler is and how sharp the point of the compass. They have to handle them very carefully. Left hand to the point, right hand to the handle, left handed students the other way around. Use a thin cardboard under the drawing in which to embed the compass point.



Exercise 14: The six-petaled rose

Start with a circle in the middle of your page with plenty of space around it. Put the point (with the same radius) on the circle line (circumference). Draw a second circle. It intersects the first circle in two points. Put the point of the compass on one of them. Draw a third circle. Go on until you get six circles around the first one. If you work carefully the last of the six will meet the first one on the line of the circle in the middle. This is a wonderful check for accuracy.

Although most wild roses have five petals, we call this a six-petaled rose or a rose of circles. Find your own title. Below you will find some examples of work from different students.

Drawing 18: Six-petaled Rose 1

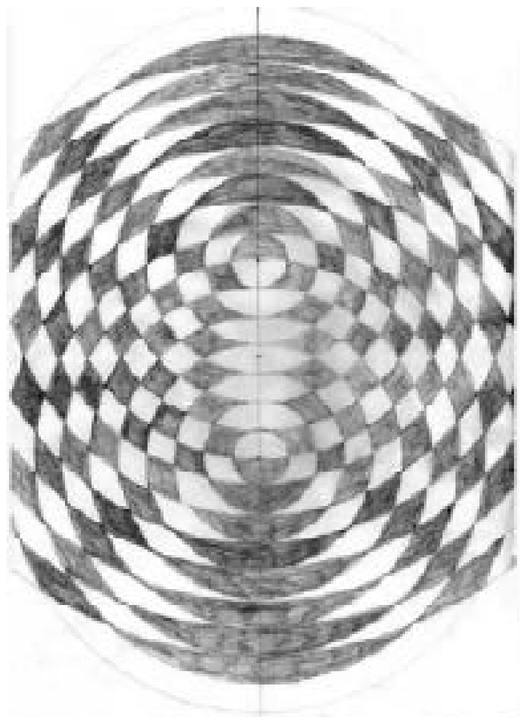
Drawing 19: Six-petaled Rose 2

Challenging line and circle exercises

Start with two intersecting lines. Open a length with the compass, e.g. half an inch. Start at the meeting point of the lines. Mark equidistant points on both lines. Join the last point on one line with the first point on the other line. Continue. Go upwards on one line and downwards on the other line. You will get a hull of a parabola.

On the right side start with a straight line. Here it is vertical. Mark the lower center. Mark equidistant points with the compass. Choose a distance of, for example, six segments for the second center. Draw the circles, always one around the first center and one around the second center. In this drawing you can find ellipses and hyperbolas. Below are student drawings.

Drawing 20: Line exercise



Drawing 21: Circle exercise

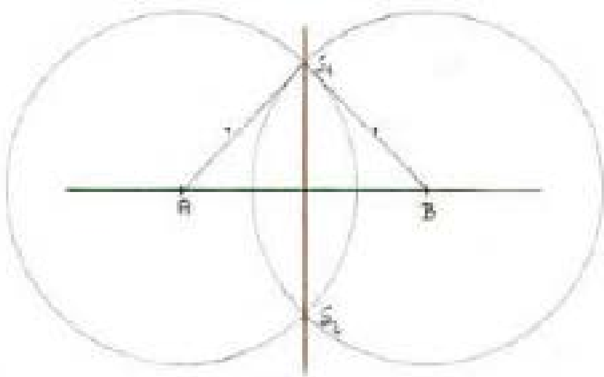
General remark

Describe the exercise in terms of a story. For example: How would you walk if there are two very dangerous beasts on both sides? The answer will be the description of the perpendicular midline of the segment joining the beasts. How can (this most important) construction be done with ruler and compass? Let the students find an answer. Let them show it on the blackboard. Ask them to describe the construction in their books. Ask them to read it. Discuss with the students whether the description is correct and complete. Be glad if they offer a very detailed description. I learned a great deal from students listening to their responses.

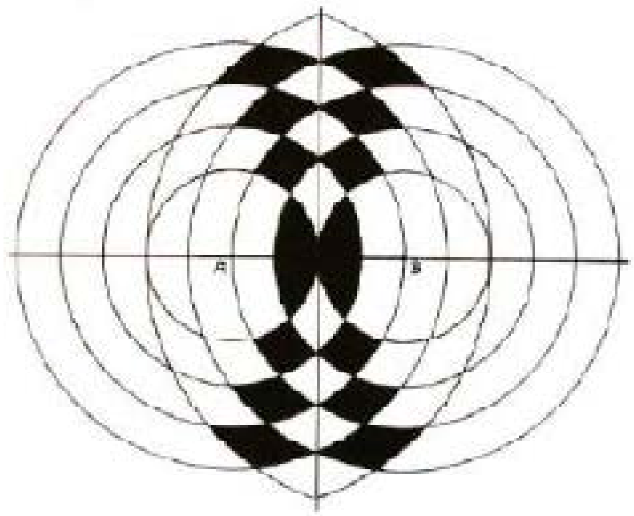
In their main lesson books should be a construction, a description, and at least one colored example of a construction where the student makes use of the fundamental construction. Below you will some examples. For more details please read also the related section in the book.

Exercise: First fundamental construction: Find the perpendicular midline of a given segment

On the left you find the construction of the perpendicular midline of the segment AB. It can be constructed with many different circles (right side). See also Fig. 76 in the book.



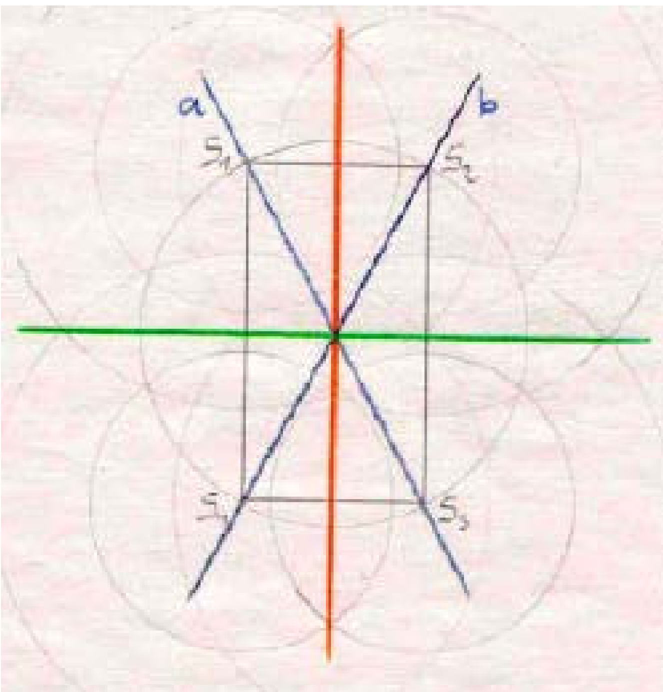
Drawing 22: Perpendicular midline 1



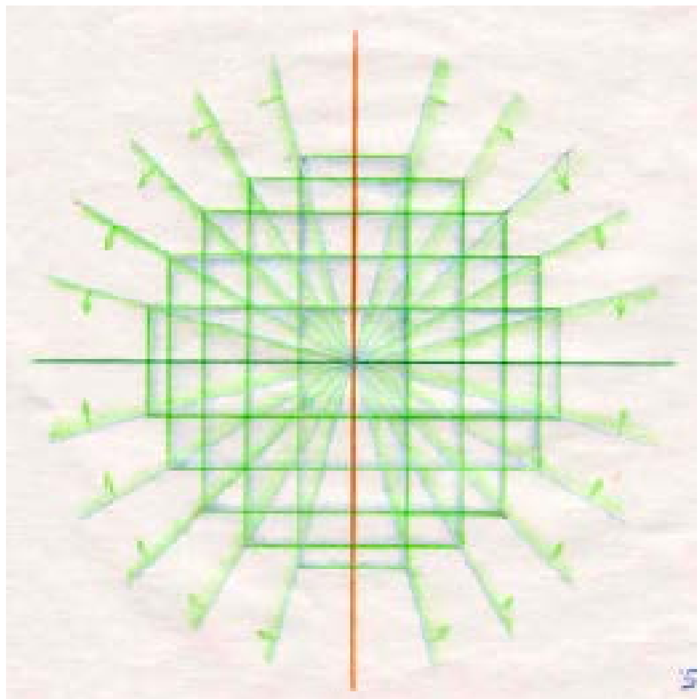
Drawing 23: Perpendicular midline 2

Fifth fundamental construction: The angle bisector

Start with two straight lines *a* and *b*. They determine four angles. To construct all bisectors you may draw a circle around the meet of *a* and *b*. The points where they meet *a* and *b* are S_1, S_2, S_3, and S_4. Use these points for the construction of the bisectors. They determine at the same time a rectangle. In the drawing on the right side you find a lot of such rectangles with the same bisectors but different lines *a* and *b*.



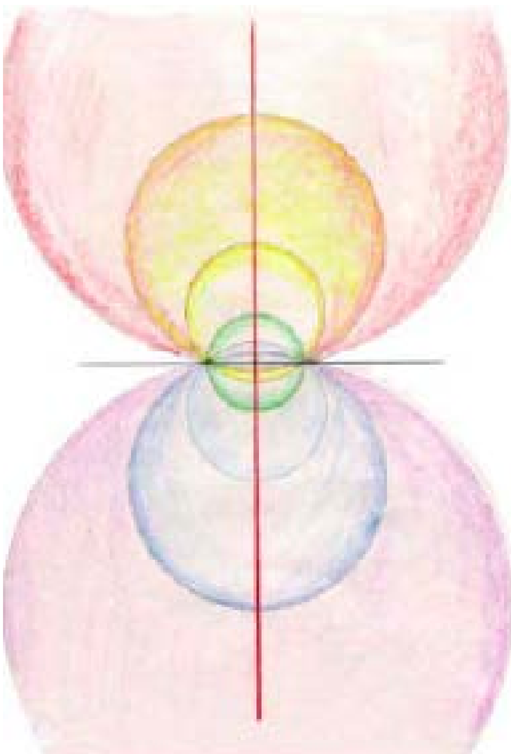
Drawing 24: Angle bisectors



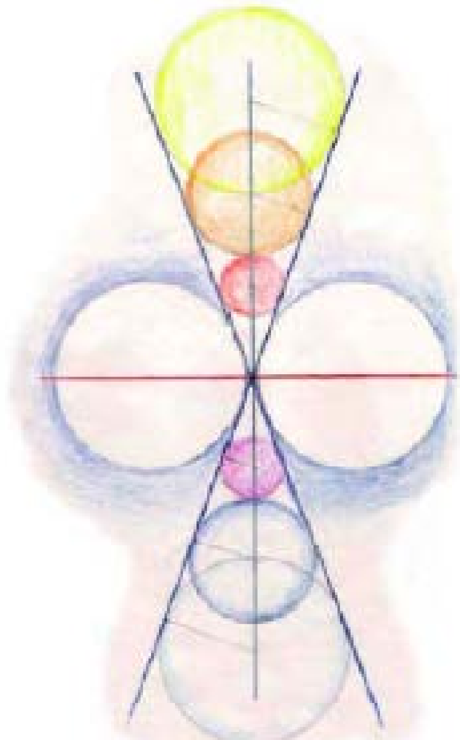
Drawing 25: Pairs of lines with the same angle bisectors

The relation between the perpendicular midline and the angle bisector

When you start with two points A and B, an infinite number of circles will pass through them. Their centers belong to the perpendicular midline of the segment AB. When you start in a polar way with two lines *a* and *b* an infinite number of circles will be tangent to them. Their centers belong to the angle bisector of the angle *ab*.



Drawing 26: Circles passing through two points

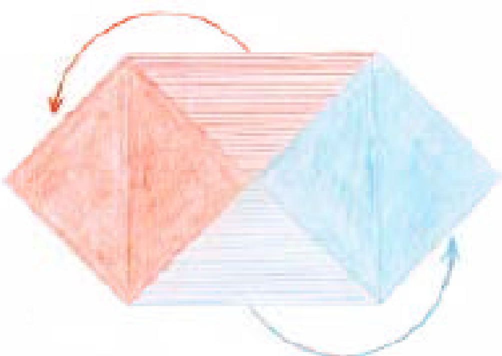


Drawing 27: Circles tangent to two lines

The first meeting with the Pythagorean Theorem

Bring a three-inch or larger cardboard square for each member of your class. Share the story from the book and then ask the students how they would cut the big square to get two smaller yet equal ones. Discuss the different suggestions and realize the best one. Draw the whole process on the blackboard. Let the students do their own cutting and drawing.

Here are two solutions. Drawings 28 and 29 explain themselves. Following Drawing 28 cut the original square along the diagonals. In Drawing 29 cut the corners off. 30 is similar (if it were folded down) to 28, but with different colors and with the original square below. With Drawing 30 you may introduce the formula $c^2 = a^2 + b^2$.



Drawing 28: Pythagorean Theorem 1

Drawing 29: Pythagorean Theorem 2

Drawing 30: Pythagorean Theorem 3

Drawing 28: Pythagorean Theorem

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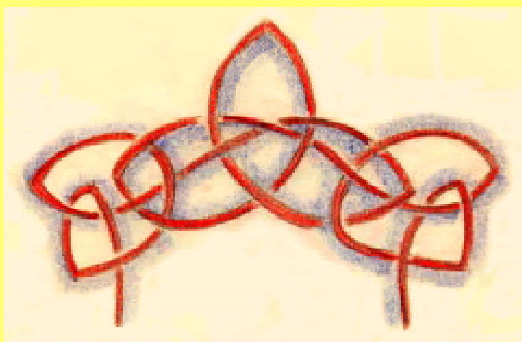
Volume 2: Freehand Form Drawing and Basic Geometric Construction in 4th and 5th Grades

Knotted Forms



by

Ernst Schuberth